[**6**] Appendix

1. Sampling Inspection

1.1 Sampling Inspection

A sampling inspection is the inspection of a small percentage of products taken from a lot (i.e., a collection of similar products, parts or materials) based on a predefined method. During the inspection, the samples are tested and the entire lot is either accepted or rejected by comparing the test results with assessment criteria. If there is no variance in lot quality characteristics, the quality of all products in the lot can be identified by picking a single sample from the lot and conducting a quality check on that sample. If there is quite a bit of variance, however, an inspection lot is constructed from production lots that were made under identical conditions so as to minimize variance.

1.2 Sampling Inspection Methods

(1) Sampling Inspection by Standards

In the sampling inspection by standards method, standards for protecting the seller and standards for protecting the buyer are developed to ensure satisfaction of both seller and buyer requirements. Seller protection is given by defining probability α, a fixed small number that indicates the probability that a good-quality lot will be rejected during inspection (producer’s risk); and buyer protection is given by defining probability β, a fixed small number that indicates the probability that a poor-quality lot will be accepted during inspection (consumer’s risk).

For example:

When producer’s risk $\alpha = 0.05$, five out of 100 good-quality lots will be rejected during inspection.

When consumer’s risk $\beta = 0.1$, ten out of 100 poor-quality lots will be accepted during inspection.

(2) Sampling Inspection by Screening

In the sampling inspection by screening method, if the sampled products pass the sampling inspection, all products are accepted as is, but if the sampled products fail the inspection, all products are inspected or “screened.” This type of method does not apply to destructive inspection, which does not permit inspection of all products.

(3) Sampling Inspection with Adjustment

This method enables rational inspections based on the use of inspection result information to date. For those lots with a good quality history, the reduced inspection is used. For those lots with a poor quality history, the tightened inspection is performed. The inspection standard is then adjusted as changes occur in the status of inspection lot acceptance. This method is defined in JIS Z9015-1.

1.3 Sampling Inspection and OC Curve

In the inspection method referred to as "sampling inspection by attributes (JIS Z 9002)," it is desirable that a good lot in which the defect rate is p0% is rendered acceptable [acceptable quality level (AQL = p0%)], and a bad lot in which the defect rate is p1% or less is rendered unacceptable.

Thus, a policy is established such that if a sample size n is taken from a lot and the number of defects found is c or less, the lot passes inspection, and if the number exceeds c, the lot fails inspection. This is referred to as the sampling inspection by attributes, and is abbreviated (n, c).

Using binomial distribution, the probability that X products will be defective in sample size n can be found using the following equation:

$$P(x) = \frac{n!}{x!(n-x)!} p^{x} (1-p)^{n-x} \qquad (x = 0, 1, 2, 3, \cdots n)$$

The graph with the horizontal axis representing the defect rate and the vertical axis representing the probability of lot acceptance, is referred to as the operating characteristic (OC) curve.

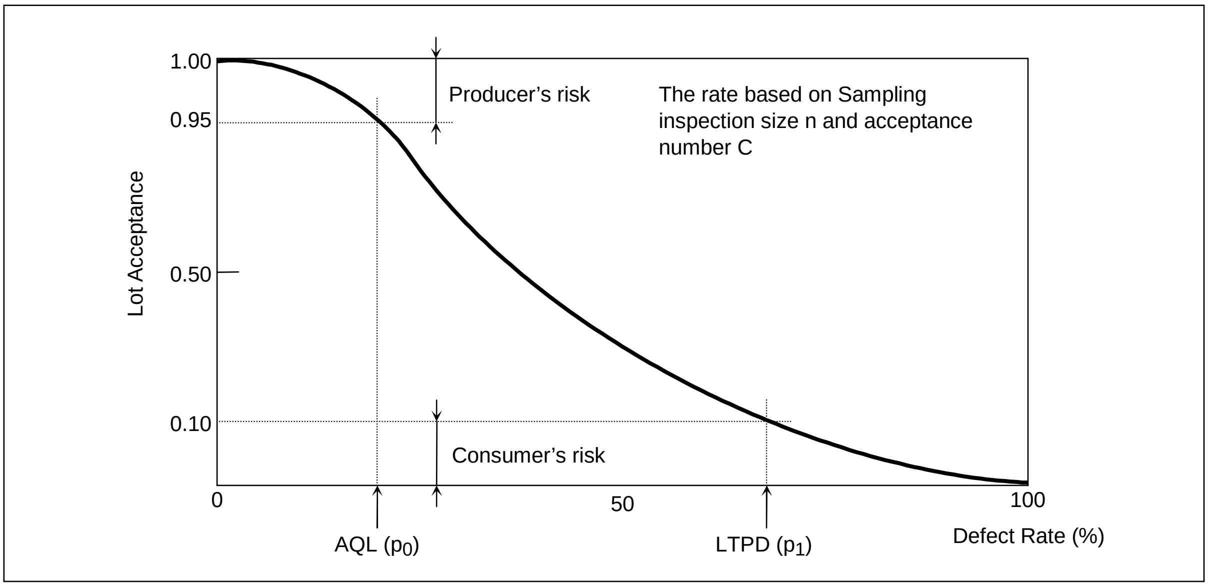


Figure 1.1 Example of an Operating Characteristic (OC) Curve

The sampling inspection by attributes, or the OC curve, is defined by the following four elements:

(1) Acceptable Quality Level (AQL): the maximum percentage of defectives (p0) in a lot considered definitely acceptable.

(2) Producer's risk (α): the probability of lots that meet the AQL will not be accepted.

(3) Lot Tolerance Percent Defective (LTPD): the minimum percentage of defectives (p1) in a lot considered definitely unacceptable.

(4) Consumer's risk (β): the probability of lots that exceed the LTPD will be accepted.

1.4 Mathematics of Sampling Inspection by Attributes

Given a sampling plan (n, c), the probability that a lot with a defect rate of p% will be accepted based on the sampling plan can be found as follows.

Suppose the probability that x defective products will appear in a sample size n is P(x). The probability of lot acceptance L(p) is the sum of the probabilities P(0), P(1), . . ., P(c-1), P(c) of 0, 1, . . . , (c-1), c defective products occurring in the sample, and can be found using the following formula:

$$L(p) = P(0) + P(1) + \cdots P(c) = \sum_{x=0}^{c} P(x)$$

Next, with sampling inspection by attributes, P(x) is calculated based on hypergeometric distribution as follows:

$$P(x) = P\,(x, n, p, N) = \frac{\binom{Np}{x}\binom{N-Np}{n-x}}{\binom{N}{n}}$$

where, N is the lot size.

However, as N increases, hypergeometric distribution and binomial distribution become approximate in value. Therefore, in practice, when N/n > 10, performing the calculation based on a simpler binomial distribution can be used.

1.5 Sampling Table

Table 1.2 shows the various inspection levels that indicate tested quantities (MIL-STD-105). If there is no particular level specified, inspection level II is normally used.

Table 1.1 Lot Size and Sample Code Letter

Lot size			Special Inspection Levels				General Inspection Levels		
			S−1	S−2	S−3	S−4	I	II	III
2	−	8	A	A	A	A	A	A	B
9	−	15	A	A	A	A	A	B	C
16	−	25	A	A	B	B	B	C	D
26	−	50	A	B	B	C	C	D	E
51	−	90	B	B	C	C	C	E	F
91	−	150	B	B	C	D	D	F	G
151	−	280	B	C	D	E	E	G	H
281	−	500	B	C	D	E	F	H	J
501	−	1200	C	C	E	F	G	J	K
1201	−	3200	C	D	E	G	H	K	L
3201	−	10000	C	D	F	G	J	L	M
10001	−	35000	C	D	F	H	K	M	N
35001	−	150000	D	E	G	J	L	N	P
150001	−	500000	D	E	G	J	M	P	Q
500001	or higher		D	E	H	K	N	Q	R

MIL−STD−105

Note: MIL-STD-105 was replaced by ANSI Z1.4.

The sample size is determined by the sample size code letter. The applicable sample size code letter is determined by the specified lot size and inspection level in Table 1.1.
With a lot size of “501 – 1200” and a general inspection level of II, the sample code is “J” and the lot acceptance quality level applied is based on the “Single Sampling Plan for Normal Inspection” (Table 1.2). Thus, from Table 1.2, based on an AQL of 0.15%, the sample size is “80,” Ac is “0,” and Re is “1.” That is, the lot acceptance quality level requires zero defects in a sample size of 80.

Table 1.2 Single Sampling Plan for Normal Inspection

Sample Cod	Sample Size	AQL (Normal Inspection)																									
		0.010	0.015	0.025	0.040	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10	15	25	40	65	100	150	250	400	650	1000
		Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re
A	2	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	30 31
B	3	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	30 31	44 45
C	5	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	30 31	44 45	↑
D	8	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	30 31	44 45	↑	↑
E	13	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	30 31	44 45	↑	↑	↑
F	20	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑
G	32	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑
H	50	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑
J	80	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑
K	125	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
L	200	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
M	315	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
N	500	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
P	800	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
Q	1250	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
R	2000	↑	↑	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑

MIL-STD-105

↓ = Use first sampling plan below arrow. If sampling size equals, or exceeds, lot or batch size, do full-lot inspection.
↑ = Use first sampling plan above arrow.
Ac = Acceptance number
Re = Rejection number

2. Mathematics of Reliability

2.1 Distributions in Reliability Analysis

The objective of any reliability study is to produce safe and reliable products. With this in mind, reliability studies can be categorized into the following three major categories:

(1) Studies of technological issues in improving reliability, (2) Studies of control issues, and (3) Studies of issues involved in evaluating the results of (1) and (2).

The evaluation and quantification of product reliability are prerequisites for selecting appropriate technical and control techniques for reliability improvement, and are necessary for determining trade-offs between reliability improvement and cost during designing as well as for assuring products.

The following is an example of an evaluation procedure in graph form.

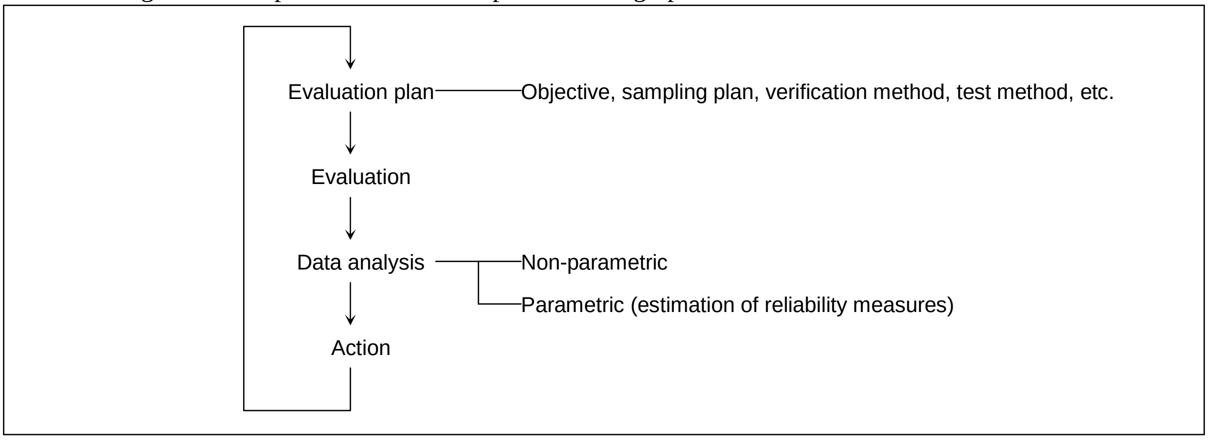


Figure 2.1 Example of Evaluation Procedure

This section describes the fundamental mathematics required for data analysis.

Data analysis is used to estimate the reliability measures (such as reliability, mean life and failure rate) previously described. The two estimation methods are the non-parametric method, which does not assume a distribution form, and the parametric method, which does assume a distribution form. The parametric method is widely used, because it is more precise and less costly, as described later. Continuous distributions (the exponential, Weibull, log-normal, normal and gamma distributions) and discrete distributions (the geometric, binomial, Poisson and negative binomial distributions) are used.

2.1.1 Continuous Distributions

(1) Exponential Distribution

The exponential distribution expresses the failure density function $f(t)$ as:

$$f(t) = \lambda e^{-\lambda t} \qquad \lambda \text{ : Failure rate (constant)}$$

The reliability R(t) is expressed as:

$$R(t) = e^{-\lambda t}$$

The failure rate (λ) is a constant, independent of time.

The mean life μ is:

$\mu = 1/\lambda$

In other words, the reciprocal of the failure rate is the mean life. The exponential distribution is characterized by the fact that the mean life and MTBF are equal and by the fact that the reliability of the surviving product after a certain time has elapsed is equal to the initial reliability of the product.

(2) Weibull Distribution

The failure density function $f(t)$ is given as:

$$f(t) = \frac{m(t-\gamma)^{m-1}}{t_o} \cdot \exp\left\{-\frac{(t-\gamma)^m}{t_o}\right\}$$

and the failure rate λ (t), the mean life μ, reliability or survival rate R(t) at time t, and cumulative failure rate F(t) at time t are expressed as follows:

$$\lambda(t) = \frac{m(t-\gamma)^{m-1}}{t_0}$$

$$\mu = t_0^{\frac{1}{m}}\,\Gamma\left(1+\frac{1}{m}\right)$$

Where, Γ = gamma function

$$R(t) = \exp\left\{-\frac{(t-\gamma)^m}{t_0}\right\}$$

$$F(t) = 1 - \exp\left\{-\frac{(t-\gamma)^m}{t_0}\right\}$$

In the above equation, m, t_0 and γ are distribution parameters. The parameter m determines the shape of the distribution and is referred to as the shape parameter. When the value of m is changed, the failure rate changes with time as shown in Figure 2.2. The distribution is exponential when m = 1. In other words, the Weibull distribution includes the exponential distribution as a special case. The failure rate increases with time when m > 1 and decreases with time when m < 1. When m is 3 or 4, the distribution is similar to the normal distribution which is described later. The parameter t_0 determines the time scale and is referred to as the scale parameter. γ determines the time at which failures start to occur and is referred to as the position parameter.

When time $(t-\gamma) = t_0^{1/m}$ is substituted in the reliability equation, F(t) = 0.632, a constant value independent of m, t_0 and γ. Therefore, $t_0^{1/m}$ is referred to as the characteristic life.

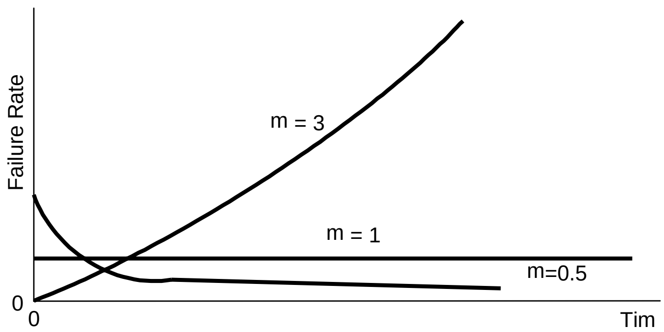


Figure 2.2 Relationship between Failure Rate and Shape Parameter m

(3) Log-Normal Distribution

In this distribution, the failure density function f(t) is expressed as:

$$f(t) = \frac{1}{\sqrt{2\pi}\sigma t}\exp\left\{-\frac{(\ln t - m)^2}{2\sigma^2}\right\}$$

This becomes a normal distribution when 1nt = y.

The mean life μ and median t_{50} are expressed respectively as:

$$\mu = \exp\left\{m + \frac{1}{2}\sigma^2\right\}$$

$$t_{50} = e^{\mu}$$

(4) Normal Distribution

In this distribution, the failure density function f(t), mean life μ and failure rate λ(t) are expressed respectively as:

$$f(t) = \frac{1}{\sqrt{2\pi}\sigma}\cdot\exp\left\{-\frac{(t-m)^2}{2\sigma^2}\right\}$$

$$\mu = m$$

$$\lambda(t) = \frac{\exp\left\{-\frac{(t-m)^2}{2\sigma}\right\}}{\int_t^{\infty}\exp\left\{-\frac{(t-m)^2}{2\sigma^2}\right\}dt}$$

where m and σ are the average and standard deviation of the distribution.

(5) Gamma Distribution

In this distribution, the failure density function $f(x)$, mean life μ and failure rate λ (t) are expressed respectively as:

$$f(t) = \frac{m^k}{\Gamma(k)} t^{k-1} \cdot e^{-mt}$$

$$\mu = k/m$$

$$\lambda(t) = \frac{t^{k-t} e^{-mt}}{\int_t^{\infty} x^{k-1} \cdot e^{-mx} dx}$$

where k is called the shape parameter. When k = 1, this distribution is similar to the exponential distribution; and when k ≥ 4, it is similar to the normal distribution.

The gamma distribution (Γ distribution) can be considered a distribution of failures occurring for the first time after k harmful shocks have been received. In this case, m is the number of harmful shocks per unit time.

2.1.2 Discrete Distributions

When it is physically impossible or inconvenient to inspect a product continuously until it fails, inspections are performed at specific intervals. In this case, time is not continuous and is treated as a discrete variable k (k = 0, 1, 2, . . .). Distributions in which time is discrete are referred to as discrete distributions.

(1) Geometric Distribution

In this distribution, the failure density function $f(k)$ is expressed as:

$$f(k) = p \cdot q^{k-1} \quad (p + q = 1)$$

where p is the probability that a failure will occur (the failure rate) during the interval from time (k – 1) to time k, and is independent of the transition of time.

The mean life μ and reliability R(k) are expressed respectively as:

$$\mu = 1/p$$

$$R(k) = q^k$$

When the time interval is made infinitely small, the result is an exponential distribution.

(2) Negative Binomial Distribution

The negative binomial distribution is a discrete form of the gamma distribution, just as the geometric distribution is a discrete form of the exponential distribution.

The failure density function $f(k)$, mean life μ and reliability R(k) are expressed respectively as:

$$f(k) = \binom{k-1}{k-m} p^m q^{k-m} \qquad \begin{matrix} m = 1, 2, \cdots \\ k\text{-}m, m+1, \cdots \end{matrix}$$

$$\mu = m \cdot q/p$$

$$R(k) = 1 - \sum_{i=m}^{k} \binom{i}{k} p^i q^{k-1}$$

In the above equations, the parameters p and m can be considered as follows: p is the number of harmful shocks per unit time interval and m is the durability against the shock. In other words, the product fails when harmful shocks are applied to the product m times.

(3) Compound Negative Binomial Distribution

In (1) and (2) above, p was constant and independent of time. If p is expressed as a function of time, as p(k), the failure density function f(k) can be expressed as:

$$f(k) = \{1-p(1)\} \cdot \{1-p(2)\} \cdots\cdots\cdots \{1-p(k-1)\} \cdot p(k)$$

$$k = 1, 2 \cdots\cdots\cdots\cdots\cdots\cdots$$

which becomes a continuous Weibull distribution when p(k) is substituted as follows:

$$p(k) = \frac{\gamma}{\beta} \left\{ k^{\beta} - (k-1)^{\beta} \right\}$$

(4) Binomial Distribution

While the geometric distribution and negative binomial distribution are used to indicate reliability, the binomial distribution and the Poisson distribution (described below) are discrete distributions mainly for sampling inspections.

The probability $P_B(r)$ of failure occurring r times during n tests is referred to as a binomial distribution, and is expressed by the equation below.

(Assuming that r failures occur when N items of the product are tested by inspecting n samples, if the relationship $10n < N$ exists, the probability of failure can be approximated by a binomial distribution.)

$$P_B(r) = \binom{n}{r} p^r (1-p)^{n-r}$$

In the above equation, p is the probability of failure occurring in a single test.

(5) Poisson Distribution

When $np = \lambda$ in the binomial distribution and

$n \to \infty$

$p \to 0$

the binomial distribution becomes a Poisson distribution with parameter λ:

$$P_B(r) = \frac{\lambda^r}{r!} e^{-\lambda}$$

where the parameter λ is equivalent to np in the binomial distribution. If:

$n > 10$

$p < 0.1$

the distribution sufficiently satisfies the Poisson distribution.

2.2 Estimating Reliability

2.2.1 Non-Parametric Estimation of Reliability Scales

As previously described, various indices such as the reliability R(t), failure distribution function F(t), failure rate λ(t) and mean life μ are used to quantify reliability, according to the situation.

Normally, each index is found after the life distribution has been identified. However, sometimes it is necessary to determine the reliability without any knowledge of the life distribution. The estimation method used in this case is referred to as the non-parametric method.

In non-parametric estimation, the indices R(t) and F(t) is expressed using the F distribution as follows:

$$\hat{R}(t) = \frac{1}{1 + \frac{r+1}{n-r} F_{\alpha}(\nu_1, \nu_2)}$$

$$\hat{F}(t) = 1 - \hat{R}(t)$$

where,

$\hat{R}(t)$ =Estimated reliability after time t has elapsed

$\hat{F}(t)$ = Estimatied cumulative failure rate up until time t

r = Number of failures that occurred during the test

n = Number of products used for the test or the number of tests performed

F = Upper α percentage point of F distribution corresponding to the variances ν_1 and ν_2

$$\nu_1 = 2n - 2r$$

$$\nu_2 = 2r + 2$$

(1–α) = Probability that the estimated reliability $\hat{R}(t)$ is equal to or greater than the true reliability. This is called the reliability level.

2.2.2 Estimating and Testing the Life Distribution Shape

(1) Estimating the Distribution Shape

The distribution shape of the life is determined by making a histogram from the data, assuming a distribution from the shape of the histogram, and then testing whether the assumption is correct. If the assumption is found to be incorrect, a different distribution is assumed and test is attempted. These steps are repeated until the correct distribution is obtained. A probability paper can be used to estimate the distribution from a histogram. Normal, log-normal and Weibull probability papers are available. The paper that gives a straight line when the data is plotted with time on the horizontal axis and cumulative failure rate on the vertical axis indicates the applicable type of distribution. (That is, normal distribution can be applied if the plot is straight on normal distribution probability paper.)

For example, let us plot a graph on Weibull probability paper using total six failures, with two failures at 1000h, one at 2000h, two at 3000h and one at 5000h for the tested 1000 products (see Figure 2.3).

The data falls approximately on a straight line and the shape parameter m is 0.7. Therefore, this distribution can be considered a Weibull distribution.

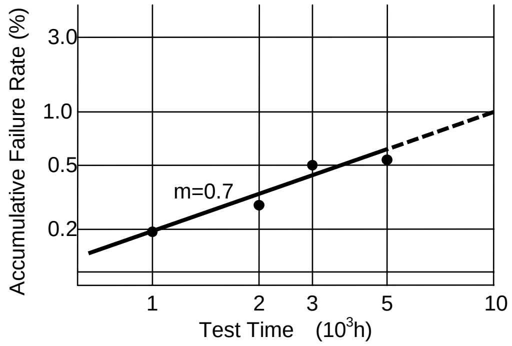


Figure 2.3 Example of Maximum Rating Continuous Operation Test

(2) Test of Distribution Shape

A method called χ^2 test is used to confirm whether the distribution of a population is equal to the estimated distribution which is based on measured values.

Assume that the failure rate is f_i in each interval $t_i - 1$ to t_i when n items of the product are tested with the test time divided into k intervals (t_1, t_2, t_3, t_4 . . . t_k). Next, the failure frequency p_i is obtained from the distribution to be tested.

When

$$x = \sum_{i=m}^{k} \frac{(F_i - p_i)^2}{p_i}$$

is substituted, if n is sufficiently large and $np_i > 10$, the distribution of x is approximated by an χ^2distribution in which the variance $\phi = k - 1$.

In order to test the assumption that the actual failure frequency occurring at each t_i is equal to the value obtained from the distribution to be verified, the value $\chi^2\alpha;\phi$ which satisfies

$$\Pr(\chi^2 \geq \chi^2(\alpha,\phi)) = \alpha$$

is determined from the χ^2 table and compared with the χ^2obtained. If

$$x \leq \chi^2(\alpha,\phi)$$

then the estimated distribution is correct.

In the above equation, α is referred to as the level of significance of the statistics. In other words, the risk that the result of test is incorrect is no more than α%. Usually a value of 5% or 10% is used.

If the distribution to be tested has m parameters, and if the parameters are estimated from data and the distribution is then tested, the variance ϕ of the χ^2 distribution is expressed as:

$$\phi = k-m-1$$

2.2.3 Parametric Estimation of Reliability Scales

When the distribution shape of the life has been identified, various indices required for reliability evaluation can be obtained by estimating the distribution parameters. The estimated parameters are themselves a function of the sampled values and form a certain distribution. The parameter values are different each time when they are sampled even from the same population. The two evaluation methods are used. One is point estimation, in which the parameters are estimated at a single point, and another is interval estimation, in which the parameters are estimated within a certain interval. An “interval estimate of confidence level γ” means that the probability that the parameter for the population exists between θ_L (lower estimate) and θ_U (upper estimate) is γ%. The confidence level is sometimes abbreviated CL.

(6) Exponential Distribution

(a) Fixed Number Testing Method

In the fixed number testing method, testing is terminated when a predetermined number of failures occurs. The parameter for the exponential distribution λ (failure rate) is expressed as follows:

$$\overline{\lambda} = \frac{r}{\sum_{i=1}^{r} t_i + (n-r)t_r}$$

$$\lambda_L = \frac{\chi^2\left(1-\frac{\alpha}{2}, 2r\right)}{2r} \cdot \overline{\lambda}$$

$$\lambda_U = \frac{\chi^2\left(\frac{\alpha}{2}, 2r\right)}{2r} \cdot \overline{\lambda}$$

where,

$\overline{\lambda}$ = Point estimate of λ

λ_L = Lower limit of λ interval estimate

λ_U = Upper limit of λ interval estimate

n = Number of tested samples

r = Total number of failures

t_i = Time when the i-th failure occurred

$\chi^2(\alpha,\phi)$ = Point where $P(\chi^2 \geq \chi^2(\alpha, \phi)) = \alpha$ in an χ^2 distribution with variance ϕ

The estimated values of the mean life μ are expressed as follows:

$$\overline{\mu} = 1/\overline{\lambda}$$

$\mu_L = 1/\lambda_U$

$\mu_U = 1/\lambda_L$

where

$\overline{\mu}$ = Point estimate of mean life

μ_L = Lower limit of mean life interval estimate

μ_U = Upper limit of mean life interval estimate

Furthermore, the point estimate of R(t) and upper and lower limits of interval estimate are expressed as:

$$\overline{R}(t) = e^{-\overline{\lambda}\cdot t}$$

$$\overline{R}_u(t) = e^{-\overline{\lambda}_L\cdot t}$$

$$\overline{R}_L(t) = e^{-\overline{\lambda}_U\cdot t}$$

(b) Fixed Time Testing Method

In the fixed time testing method, the test is terminated at a predetermined time t_c regardless of the number of failures. The point estimate and interval estimate of λ are expressed as follows:

$$\overline{\lambda} = \frac{r}{\sum_{i=1}^{r} t_i + (n-r)\cdot t_c}$$

$$\lambda_L = \frac{\chi^2\left(1-\frac{\alpha}{2}, 2r+2\right)}{2r}\cdot\overline{\lambda}$$

$$\lambda_U = \frac{\chi^2\left(\frac{\alpha}{2}, 2r+2\right)}{2r}\cdot\overline{\lambda}$$

(2) Normal Distribution

There are two normal distribution parameters: μ and σ^2. Parameter μ is the mean life and σ^2 is the variance of the distribution. The point estimates of these values are expressed as follows:

$$\overline{\mu} = \frac{\sum_{i=1}^{n} t_i}{n}$$

$$\overline{\sigma}^2 = \frac{\sum_{i=1}^{n} (t_i - \overline{\mu})^2}{n-1}$$

The upper limit μ_U and lower limit μ_L of the mean life within the reliability interval are:

$$\mu_U = \overline{\mu} + t(\alpha, n-1)\cdot\sqrt{\frac{\overline{\sigma}^2}{n}}$$

$$\mu_U = \overline{\mu} - t(\alpha, n-1)\cdot\sqrt{\frac{\overline{\sigma}^2}{n}}$$

And the upper limit σ_L^2 and lower limit σ_U^2 of the variance σ^2 within the reliability interval are:

$$\sigma_L{}^2 = \frac{(n-1)\overline{\sigma}^2}{\chi^2\left(1-\frac{\alpha}{2}, n-1\right)}$$

$$\sigma_U{}^2 = \frac{(n-1)\overline{\sigma}^2}{\chi^2\left(\frac{\alpha}{2}, n-1\right)}$$

where

$t(\alpha,n-1)$ = The value of t for which $P(t > t(\alpha,n-1)) = \alpha$ in the t distribution table

$\chi^2\left(\frac{\alpha}{2}, n-1\right) = \chi^2$ for which $P\left(\chi^2 \geq \chi^2\left(\frac{\alpha}{2}, n-1\right)\right) = \frac{\alpha}{2}$ in the χ^2 distribution table

(3) Weibull Distribution

The Weibull distribution has three parameters: m, t_o and γ, and it is very difficult to analyze data by calculation. Hence, the Weibull probability paper is often used for estimation. If m is known and $\gamma = 0$, t_o is expressed as follows:

$$\bar{t}_0 = \frac{\sum_{i=1}^{r} t_i{}^m + (n-r)t_r{}^m}{r}$$

(4) Log-Normal Distribution

Similar to normal distribution, there are two parameters μ and σ^2. The point estimates are expressed as follows:

$$\overline{\mu} = \frac{\sum_{i=1}^{n} \ln t_i}{n}$$

$$\overline{\sigma}^2 = \frac{\sum_{i=1}^{n} \left(\ln t_i - \overline{\mu}\right)^2}{n-1}$$

And the estimate $\overline{M}$ of the mean life is:

$$\overline{M} = \exp\left(\overline{\mu} + \frac{\sigma^2}{2}\right)$$

2.2.4 Using Probability Papers

Distribution studies using a probability paper are very simple and require no complicated calculations. The method is widely used to verify distribution parameter theories and to find distribution parameters.

Various types of probability paper are available and their use is widely known. Described below are a few tips on how to plot data on probability paper and determine whether the result is a straight line.

Many plotting methods have been devised for processing data on probability paper and making estimates as accurate as possible. The plot for a product for which n samples have been tested and for which the i-th product was defective is (t_i, F_i), where t_i is the time after which product number i failed and F_i is the cumulative failure rate.

The following values are generally used to plot F_i:

(1) i/n

(2) (i–0.5) / n

(3) (i–1) / (n–1)

(4) i/(n + 1)

(5) $(i-\alpha_i) / (n-\alpha_i-\beta_i + 1)$

(1) to (4) are very simple, but the last datum (that is the nth datum) is not utilized in (1) and the first datum is not utilized in (3). Therefore, (2) or (4) is recommended.

Method (5) has been devised as an improvement over (4): $\alpha_I = \beta_I = 3/8$ for the normal distribution; $\alpha_i = 0.52 (1-1/m)$ and $\beta_i = 0.5-0.2 (1-1/m)$ for the Weibull distribution with shape parameter m.

When determining the straightness of the curve, the conventional least square method can be used. However, the data will not be distributed evenly and the variance will be smallest around the central part of the curve. Thus, when drawing the curve, make sure that it adheres closely to these central points.

2.3 Relationship between Failure Models and Life Distributions

2.3.1 Rope Model

The previous section described the mathematical methods for estimating the life distribution. The life distribution can be further narrowed down if the relationship between the life distribution and failure is known. When viewed from this perspective, the exponential distribution can be considered as a distribution of products which fail when randomly subjected to m harmful shocks per unit time. Similarly, the gamma distribution can be thought of as the case where a product receives k shocks before it fails.

Now assume that a product consists of many components, just as a rope consists of many strands. A rope fails when all of its strands are cut. Therefore, the following relationship exists between the reliability of a product and the reliability of its components:

$$R_D = 1 - \mathop{\pi}_{i=1}^{k} (1 - R_i)$$

where R_D is the reliability of the product, and R_i is the reliability of the nth component, and k is the number of components.

Given that the distribution shape of the life for each component forms an independent exponential distribution with the same shape, the distribution for the product as a whole is a gamma distribution with a shape parameter k and a scale parameter m that is the same m value as the exponential distributions for the components.

A product that will fail only when all of its components fail is referred to as a “rope model” or “parallel model.” This model is used to study the problem of product fatigue and redundancy in design. When k in a gamma distribution becomes large, the distribution becomes similar to a normal distribution and the mean value becomes equal to k/m. Therefore, a normal distribution can be considered an extreme case of the rope model.

2.3.2 Weakest Link Model

In contrast to the rope mode, a model similar to a chain of k links, where the failure of the weakest link results in the failure of the entire chain, is referred to as the "weakest link model." This also applies to an article of equipment consisting of k components where the failure of any single component results in the failure of the equipment as a whole. For this reason, the weakest link model is also referred to as the "serial model." In this case, the following relationship exists between the reliability R_D of the product and the reliability R_i of the components.

$$R_D = \underset{i=1}{\overset{k}{\pi}} R_i$$ k: Number of components

The Weibull distribution is one of the distributions that represent the weakest link model. In addition, the following double-exponential distribution, an extreme case of the Weibull distribution, is also used to represent the weakest link model.

$$F(t) = 1 - \exp\left\{-\exp\left(\frac{t}{n}\right)\right\}$$

2.3.3 Proportional Effect Model

Given that $X_1 < X_2 < X_3 < \ldots. < X_4$ are the fatigue cracks at each phase, the size of the fatigue crack at each phase is proportional to that of the previous phase. That is, if the relationship below exists, then the distribution of X_n is a log-normal distribution.

$X_i = \alpha_i X_{i-1}$

αi = Constant

i = 1, 2, ……

2.3.4 Stress and Strength Model

In this type of model, a product fails when stress accumulates beyond its strength. In this model, failure can be calculated as the overlap of the stress distribution and the strength distribution.

If stress and strength are both normal distributions, the life distribution will also be a normal distribution. If the average stress at a given time is μ_s and the standard deviation is σ_s, and similarly for the strength distribution, if μ_k is the average and σ_k is the standard deviation, then the level of unreliability represented by the area of normal strength distribution in which the strength is below zero, the average is equal to $(\mu_k - \mu_s)$ and the standard deviation is equal to $\sqrt{\sigma^2 k + \sigma^2 s}$.

2.3.5 Reaction Theory Model

This model attempts to estimate life using a failure physics method. It assumes that a failure is caused at a microscopic level, where changes at the atomic and molecular levels cause harmful reactions and result in failure when the changes reach a certain threshold. The following stress and life relations based on the Arrhenius model of chemical reactions are widely used.

$$\ln L = A + \frac{B}{T} - \alpha \ln S$$

L = Mean life

A, B, α = Constants

T = Temperature (°K)

S = Stress other than temperature

2.3.6 Reliability Model for Equipment

(1) Serial Model

For an article of equipment consisting of n components, if the equipment fails when one of its components fails, the reliability $R_s(t)$ of the equipment can be expressed as a function of the reliability of each component $R_i(t)$ as follows:

$$R_s(t) = 1 - \underset{i=1}{\overset{n}{\pi}} R_i(t)$$

(2) Parallel Model

For an article of equipment consisting of n' components running in parallel, with the equipment continuing to function as long as any of the parallel components is still running, the following is true:

$$R'_s(t) = 1 - \underset{i=1}{\overset{n'}{\pi}} (1 - R_i(t))$$

In this case, the reliability is better than that of equipment consisting of only one component.

[Bibliography]

Reliability Handbook, edited by W. G. Ireson (McGraw Hill)

Introduction to Reliability Engineering, by H. Shiomi (Maruzen)

3. Derating Concepts and Methods

3.1 Concepts of Derating

Reliability greatly changes based on the level of derating, even within rated values.

In particular, the usage conditions of discrete semiconductors such as power devices are left to system designers; they affect the mean time between failures (MTBF) of a system and its useful life span. Therefore, system designers must consider the reliability characteristics of semiconductor devices and check their individual derating factors. Temperature and power derating factors particularly need be modeled with environmental conditions.

Here is an example of derating. These conditions include those for worst case scenarios, including surges.

- Temperature: Tj = 80% Tj MAX or less
  *Assuming approximately 10 years of intermittent use (about three hours per day)
  Tj = 50% Tj MAX or less
  *Assuming approximately 10 years of 24-hour use in applications that require high reliability
  * Tj should be replaced by a temperature rating specified in the datasheet (e.g., Ta or Tch) if the device is not rated with Tj.
- Voltage: 80% of the maximum rating or less (Integrated circuits: within the recommended operating range)
- Current: Average current 80% of the maximum rating or less (rectifying elements: 50% of the maximum rating or less)
- Peak current 80% of the maximum rating or less
- Power: 50% of the permissible maximum loss

For high reliability, a further derated design is required. Derating factors should be determined, referring to the technical datasheets of individual devices and their reliability data.

In addition, there are safe operation areas (SOAs) and other factors that must be observed during product design. Therefore, be sure to follow the specifications of each individual component.

It is no exaggeration to say that the reliability of an end product depends on that of its constituent components. To realize high reliability, an appropriate derating design is required, considering the reliability characteristics of semiconductor devices.

3.2 Methods of Derating

One well-known method for estimating the failure rate of a semiconductor device is described in the MIL standard MIL-HDBK-217. In this method, the failure rate of a semiconductor element with respect to operating conditions is found using a failure rate prediction model based on past field data.

Figure 3.1 shows an example of a derating curve of a transistor (low frequency, bipolar) based on the MIL standard.

This prediction method, however, classifies semiconductor devices into large groups according to application without reflecting the used technical levels and process stability, which differ for each actual device. To compensate for this, semiconductor manufacturers can use a similar failure rate derating curve based on independently accumulated field data and in-house acceleration test results, although this method is not as detailed as the MIL standard method.

Figure 3.2 shows a derating curve of a transistor that is based on acceleration tests and field performance results obtained by Toshiba.

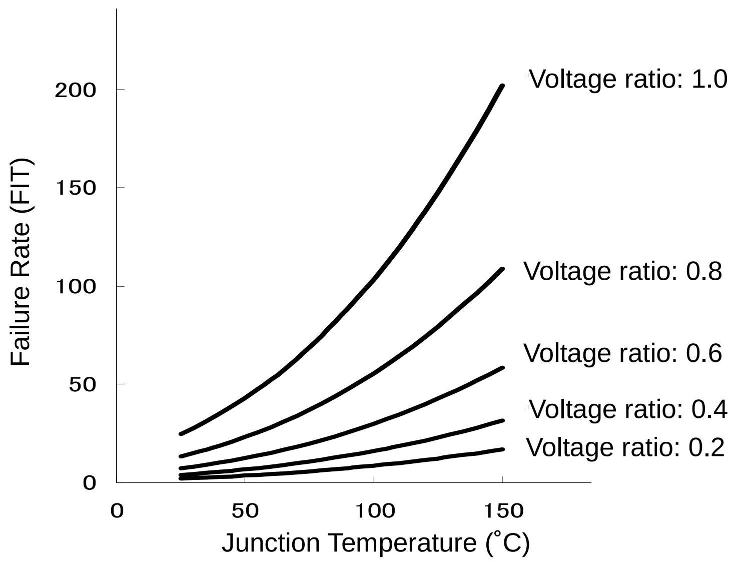


Figure 3.1 MIL-HDBK-217-Based Derating

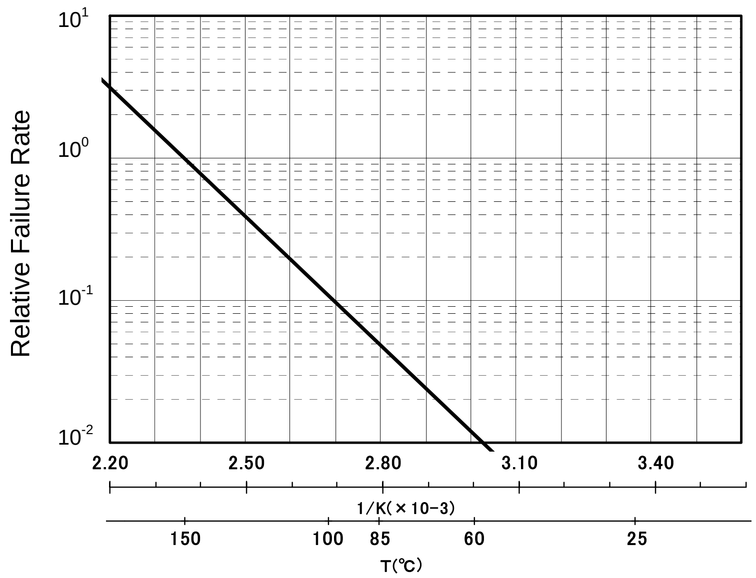
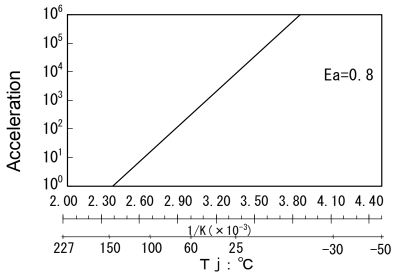


Figure 3.2 Transistor Derating Curve

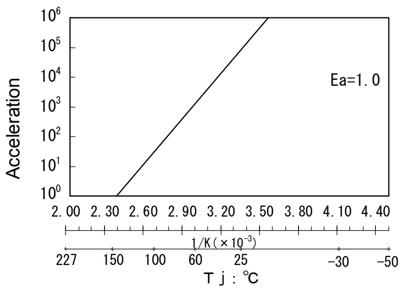
It has been verified that Toshiba’s semiconductor devices do not enter the wear-out failure period of the bathtub curve within the reliability test time. For system design, the ratings of semiconductor devices must be derated so that they will not reach the wear-out failure period (within the reliability test time) in each failure mode under the assumed actual usage conditions.

The following shows how to calculate reliability test conditions for commonly used failure modes, based on the results of Toshiba’s past reliability testing.

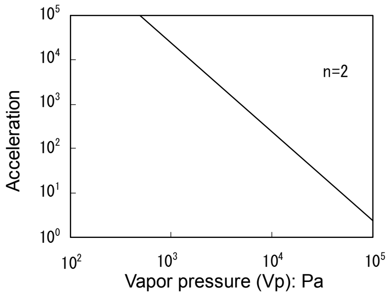
(1) Characteristics variations (Vth, hFE, etc.)

Acceleration model	Arrhenius model: $\alpha \propto \exp(-Ea/k \cdot 1/T)$ Ea : Activation energy (0.8 eV) k : Boltzmann's constant T : Absolute temperature (in Kelvin) 
Reliability test conditions	High-temperature operation test or high-temperature bias test; e.g., Tj = 150°C, 1000h
Assumed condition examples	Only the On periods are assumed. Tj = 120°C, 10h • • • (a) Tj = 90°C, 2000h • • • (b) Tj = 60°C, 48000h • • • (c)
Conversion	The assumed conditions can be converted to reliability test conditions as follows (e.g., Tj = 150°C): (a): $\exp\{-0.8/8.617 \times 10^{-5} \cdot (1/393 - 1/423)\} \times 10h = 1.9h$ (b): $\exp\{-0.8/8.617 \times 10^{-5} \cdot (1/363 - 1/423)\} \times 2000h = 53.2h$ (c): $\exp\{-0.8/8.617 \times 10^{-5} \cdot (1/333 - 1/423)\} \times 48000h = 127.6h$ (a) + (b) + (c) = 182.7h

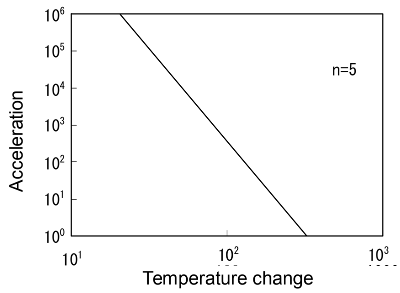
(2) Metal interconnect faults (Delamination of bonding balls due to the growth of Au-Al alloy)

Acceleration model	Arrhenius model: $\alpha \propto \exp(-Ea/k \cdot 1/T)$ Ea : Activation energy (1.0 eV) k : Boltzmann's constant T : Absolute temperature (in Kelvin) 
Reliability test conditions	High-temperature storage test; e.g.,Tj = 150°C, 1000h
Assumed condition examples	Both the On and Off periods are assumed. Tj = 120°C, 100h • • • (a) Tj = 90°C, 7000h • • • (b) Tj = 60°C, 80000h • • • (c)
Conversion	The assumed conditions can be converted to reliability test conditions as follows (e.g., Tj = 150°C): (a): $\exp\{-1.0/8.617 \times 10^{-5} \cdot (1/393 - 1/423)\} \times 100h = 12.3h$ (b): $\exp\{-1.0/8.617 \times 10^{-5} \cdot (1/363 - 1/423)\} \times 7000h = 75.2h$ (c): $\exp\{-1.0/8.617 \times 10^{-5} \cdot (1/333 - 1/423)\} \times 80000h = 48.3h$ (a) + (b) + (c) = 135.8h

(3) Metal interconnect faults (Al corrosion)

Acceleration model	Absolute vapor pressure model: $\alpha \propto Vp^{-n}$ Vp : Absolute vapor pressure n : Acceleration factor (n = 2 is used.) 
Reliability test conditions	High-temperature/high-humidity storage test or high-temperature/high-humidity bias test; e.g., Ta = 85°C, RH = 85%, 1000h
Assumed condition examples	Both the On and Off periods are assumed. Ta = 30°C/RH = 85% (Vapor pressure: 3608.2Pa), 18000h • • • (a) Ta = 20°C/RH = 70% (Vapor pressure: 2376.2Pa), 55000h • • • (b) Ta = 10°C/RH = 65% (Vapor pressure: 797.8Pa), 18000h • • • (c) * When the test equipment is on, an increase in temperature of the equipment causes the relative humidity to be decreased. It is assumed, however, that the vapor pressure remains unchanged. * The vapor pressure data is an excerpt from Chronological Scientific Tables compiled by the National Astronomical Observatory of Japan.
Conversion	The assumed conditions can be converted to reliability test conditions as follows (e.g., Ta = 85°C/RH = 85% (vapor pressure: 49146.2 Pa)): (a): $(49146.2/3608.2)^{-2} \times 18000h = 97.0h$ (b): $(49146.2/2376.2)^{-2} \times 55000h = 52.6h$ (c): $(49146.2/797.8)^{-2} \times 18000h = 5.5h$ (a) + (b) + (c) = 155.1h

(4) Faults caused by repetitive thermal stress (package cracks, degradation of die bond)

Acceleration model	Eyring model: $\alpha \propto \Delta T^{-n}$ ΔT : Temperature change n : Acceleration factor (n = 5 is used.) 
Reliability test conditions	Temperature cycling test; e.g., 100 cycles between –55°C and 150°C (Ta)
Assumed condition examples	ΔT = 60°C, 7950 cycles • • • (a) (Temperature change of the device itself and the ambience from 20°C to 80°C when the test chamber is switched on) ΔT = 15°C, 2650000 cycles • • • (b) (Temperature change of the ambience from 80°C to 95°C when the test chamber is on)
Conversion	The assumed conditions can be converted to reliability test conditions as follows (e.g., ΔT = 205°C): (a): $(205/60)^{-5} \times 7950$ cycles = 17.1 cycles (b): $(205/10)^{-5} \times 2650000$ cycles = 5.6 cycles (a) + (b) = 22.7 cycles

[Bibliography]

- MIL-HDBK-217
- JEITA EDR-4704, “Application guide of the accelerated life test for semiconductor devices”